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## A QUASI-THREE-DIMENSIONAL GROUND MODEL FOR EARTHQUAKE RESPONSE ANALYSIS OF UNDERGROUND STRUCTURES

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# A QUASI-THREE-DIMENSIONAL GROUND MODEL FOR EARTHQUAKE RESPONSE ANALYSIS OF UNDERGROUND STRUCTURES

――地盤モデルの構成―

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### 1. Introduction

In the earthquake response analysis of a shield tunnel and a submerged tunnel, one-dimensional spring-mass system proposed by Tamura<sup>1)</sup> has been widely used as the ground model. The model was verified by vibaration tests on the ground models including tunnels. However, the application of the model is only appropriate to the ground where the ground condition is considered to be uniform in the direction perpendicular to the line on which response analysis is performed.

A shield tunnel, which is one of the representative tunnels in urban area in Japan, is usually constructed in alluvial soft ground. The topography of alluvium is generally complicated and bounded by diluvium irregularly. In order to grasp dynamic behaviors of the alluvial ground, it is necessary to take these factors into consideration, but much cost and time and laborious works are needed. So that, Tamura proposed a simple two-dimensional spring-mass system as a mathematical model in 1983<sup>20</sup>. In the system, however, there was a problem in representing Poisson's ratio of the ground soil.

The authors have been conducting earthquake observations of shield tunnels in such alluvial ground and found out that the three-dimensional ground structure is one of the major factors governing behaviors of shield tunnels during earthquakes<sup>3)</sup>. In this paper, the mathematical model of ground

for the analysis of dynamic bahaviors of under-

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ground structures will be introduced for practical use. The new model is a developed one from Tamura's Model and only the fundamental shear vibration is taken into account as the former model does. However, it can easily deal with three –dimensional geological structures. The characteristics of the model is the fact that it is a composite model of FEM and spring-mass system. By introducing FEM, the spring constants of the springs connecting the lumped masses can be reasonably determined and the mechanical condition of the ground is adequately expressed.

#### 2. A Quasi-Three-Dimensional Ground Model

In the first step, we consider a rectangular ground element J shown in Fig.1. At nodal point i, the constant  $(EF)_i$  is defined in the followings :

$$(EF)_{i} = \int_{o}^{H_{i}} E'_{i}(z) F_{i}(z) dz$$
 (1)

where,  $H_i(z)$ ; Thickness of surface layer at point i

- $E'_i(z)$ ; Vertical distribution of Young's modulus at nodal point i
- $F_i(z)$ ; Horizontal displacement function at nodal point *i* defined by equation (2)

z ; Depth

$$F_i(z) = \frac{f_i(z)}{\int_0^{H_i} \frac{m'_i(z)f_i(z)dz}{m'_i(z)dz}}$$
(2)

where, 
$$m'_i(z)$$
; Soil mass of unit cross-section at  
point *i*



Fig.1 A Rectangular Ground Element J and Its Composition

 $f_i(z)$ ; Modal vector of the fundamental

shear vibration at point i $F_i(z)$  means the horizontal movements of soil column i along the depth and normalyzation is carried out as shown in equation (2) so that  $F_i(z)$ may equal to 1.0 at the depth corresponding to the center of the inertia force of the soil column. The equivalent Young's modulus  $(EF)_i$  in equation (1) is calculated by the integration of  $E'_i(z) \cdot F_i(z)$  in the direction of depth. Therefore, the soil column can be transformed to the plate element with unit thickness and with Young's modulus of  $(EF)_i$  at point i. As the values of  $(EF)_i$  are given by equation (1) at four nodal points of an element, Young's modulus  $E_j$  of plate element J can be calculated as follows :

$$E_{J} = \frac{\sum_{i=1}^{4} (EF)_{i}}{4} \tag{3}$$

In the same manner, Poisson's ratio of plate element *J* is determined as follows :

$$V_{J} = \frac{\sum_{i=1}^{J} v_{i}}{4} = \frac{1}{4} \sum_{i=1}^{4} \frac{1}{\int_{0}^{H_{i}} F_{i}(z) dz} \int_{0}^{H_{i}} F_{i}(z) v'_{i}(z) dz (4)$$

where,  $V_J$ ; Poisson's ratio of plate element J

 $v'_i(z)$ ; Poisson's ratio of nodal point i

*v<sub>i</sub>* ; Average Poisson's ratio at nodal point *i* 

The applicability of equation (3) and (4) is limited to the case that there is no large difference in dynamic characteristics among four nodal points. If the shape of an element is fairly different from rectangular shape, the figure 4 – denominator in equations (3) and (4) must be replaced by other figures, taking the relative locations of nodal points into consideration. When the shape of an element is triangular, for example, the denominators in equations (3) and (4) are 3.

In the next stage, the soil mass at nodal point i should be determined.

Defining that the mass of soil column at point i is  $m_i$ , the soil mass concentrating on nodal point i is given by integrating  $m_i$  over the influence area  $(AREA)_i$  shown in Fig. 2.

$$M_i = m_i \cdot (AREA)_i \tag{5}$$

Thus, spring constant  $K_3$ , which is the spring constant between the base ground and the soil mass, is calculated using the fundamental period of the soil column and soil mass  $M_i$  at point *i*.

$$K_{3,i} = M_i \cdot (\frac{2\pi}{T_i})^2 \tag{6}$$

Fig.3 illustrates a drowned valley sedimented by alluvial soil and its simple modelization by the proposed method. As shown in Fig.3, alluvial layer is substituted for springs  $K_{3,i}$ , soil masses  $M_i$  and a plate with material properties  $E_J$  and  $V_J$ . Using  $E_J$ and  $V_J$  of the plate element J, it is possible to form rigidity matrix  $[K_2]$  connecting soil masses by finite element mehtod. The stresses of the soil plate are analyzed under plane-stress condition in this model.

The equation of motion is as follows:



Fig.2 Soil Mass  $M_i$  and Influence Area  $(AREA)_i$  at Nodal Point i



(a) A Drowned Valley Surrounded by Diluvium



Fig.3 A Schematic Representation of the Proposed Method to Modelize Alluvial Ground

 $[M] \cdot [\overset{\vec{X}}{Y}] + [C] \cdot [\overset{\vec{X}}{Y}] + [K] \cdot [\overset{\vec{X}}{Y}] = -[M_e] \cdot [\overset{\vec{U}}{W}] \quad (7)$ 

- where, [M]; Mass matrix composed of  $M_i$ 
  - [C]; Damping matrix
  - [K]; Rigidity matrix, which is the sum of  $[K_2]$  and  $[K_3]$
  - X, Y; Displacements of soil masses in xand y direction, respectively
  - $\ddot{U}, \ddot{W}$ ; Accelerations of base ground in x and y direction, respectively

 $[M_e]$ ; Effective mass matrix

Effective mass matrix is composed of effective soil masses  $M_{e,i}$ . And  $M_{e,i}$  is given by the following equation:

$$M_{e,i} = (AREA)_i \frac{(\int_0^{H_i} m'_i(z)f_i(z)dz)^2}{\int_0^{H_i} m'_i(z)f_i^2(z)dz}$$
(8)

Using the equations mentioned above, the behavior of the ground during earthquakes based on the fundamental shear vibration, which may majorly influences on underground structures during severe

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tremors, can be analyzed. The ground displacements transmitted to the underground structure are easily calculated as the product of the ground displacement at nodal point i ( $X_i$  and  $Y_i$ ) and horizontal displacement function at nodal point i( $F_i(z)$ ).

#### Conclusions

The authors presented a quasi-three-dimensional ground model for earthquake response analysis of underground structures. The characteristic features of the model are as follows :

- (1) It can deal with three-dimensional geological structures.
- (2) It is a composite model of FEM and spring -mass system.
- (3) By the introduction of FEM, mechanical conditions connecting lumped masses (soil masses) can be reasonably determined with the assump-

tion of plane-stress condition.

The authors will report the results of the verification of the model, comparing the analysis and vibration tests using shaking table, to the next issue of "SEISAN-KENKYU".

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